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Mathematics Computation: Helping Handout for School

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INTRODUCTION

A significant challenge for teachers is how to balance the performance of automatic computations with equally important flexibility and understanding. Computation—producing a correct answer for a calculation—can be thought of as one component of a larger ability that many mathematics educators refer to as *procedural fluency*, which is defined as "skill in carrying out procedures flexibly, accurately, efficiently, and appropriately" (Kilpatrick, Swafford, & Findell, 2001, p. 121). On the one hand, it is essential for students to learn to automatically execute computations by recalling number facts quickly and effortlessly. On the other hand, we also want students to be able to think critically about their computational strategies, including why the strategies work, whether a particular strategy is the best choice, and whether there are alternative strategies that might be preferable. Teachers can help students balance these seemingly competing aims by helping them perform math computations fluently, but also with flexibility and understanding. The recommendations in this handout are designed to help teachers achieve that aim. The strategies should be particularly helpful when working with students who struggle with math computation, including those with a specific learning disability in math computation.

WHAT TO CONSIDER WHEN SELECTING INTERVENTIONS AND SUPPORTS

Students face many learning challenges in pursuit of procedural fluency. Teachers' choice of supports and interventions, as well as their general instructional approach, depends largely on which of the following challenges are most in need of being addressed:

- *Lack of automaticity.* The student may face challenges with rapidly and automatically recalling number facts (such as sums or products), which limits the ability to compute quickly.
- *Limited memory for and repertoire of procedures.* The student may face challenges with remembering which steps in a procedure to execute and in what order. Additionally, if the student does not know multiple methods for executing a computation, does not know why these methods work, and is not able to choose or implement appropriate methods, then the student cannot make strategic choices about which method best aligns with a problem.
- *Limited number sense.* If the student does not have a sense of numbers and their magnitude, including the ability to estimate, then he or she is likely to struggle with analyzing the work, such as predicting an answer or evaluating whether an answer is reasonable.
- *Limited understanding of meanings underlying procedures, such as understanding of symbols*. Incorrect calculations may result from misunderstandings about mathematical features in a problem, such as having limited interpretations for symbols that represent operations or equivalence. When a student repeatedly makes a particular error during problem solving, it may be the case that the student holds an incorrect or weak understanding of an underlying mathematical principle or feature.
- *Affective and motivational considerations*. If the student has negative emotional states that arise from difficulties with procedural fluency, including math anxiety and low confidence in his or her mathematics skill, or lacks motivation to exert effort, these emotional and motivational states are

likely to interfere with the student's execution of computations.

In selecting appropriate instructional supports and interventions, teachers should attempt to diagnose which of the above challenges the student seems to be facing. Diagnostic testing can be either formal, with the support of a school psychologist, or informal, through formative assessment in the classroom. In the process of assessment, it is important to consider the alignment of the interventions and the teacher's instructional approach with the curriculum and the culture of the classroom. For example, do the recommended methods for teaching computation align well with the methods the teacher uses during instruction and as presented in the student's textbook? Do the explicit and implicit priorities established by the instructional intervention mesh well with the classroom? Does the intervention reward speed and accuracy only? If so, this could result in challenges for a student who has been encouraged to value explanation and justification. Does the intervention emphasize product over process, with greater focus on getting correct answers rather than on careful selection of strategies? If so, this may cause difficulties for a student who has been encouraged to develop conceptual understanding and use multiple methods.

RECOMMENDATIONS

To address learning challenges related to mathematics computation, teachers can consider the following instructional supports and interventions.

1. ***Provide the student with explicit instruction about how to execute procedures for specific problems.*** Explicit instruction has two main components. In the first, the teacher demonstrates a specific step-by-step strategy for solving a particular problem. In the second, the student practices using the particular strategy after the teacher's demonstration. Explicit instruction also can be provided using fully or partially worked-out examples. Within the examples, the student is shown a strategy to solve a specific problem and is given opportunities to practice that strategy in similar problems. To be effective, explicit instruction must include clear teacher models of computation, opportunities for guided practice, and regular feedback.
2. ***Teach the student to use heuristics that may be helpful when solving any mathematics problem.*** Instruction in heuristics is related to but distinct from explicit instruction in that students are taught general strategies that are not problem-specific. For example, heuristic instruction could include giving students steps such as, "Read the problem. Highlight key words. Solve the problem. Check your work." When teaching heuristics, encourage the student to talk through the solutions and reflect on the decisions made. The class as a whole also should be encouraged to use multiple heuristics. That is, encourage each student to select a strategy and then help the class discuss and reflect on the students' different strategy selections. This should be followed by assisting the students with using the strategies discussed. Although this approach can be effective, be careful not to overwhelm students with learning disabilities, who often struggle with keeping information in memory. Provide the necessary scaffolding and feedback to aid those students through the reflection and execution process.
3. ***Increase the student's automaticity and flexibility through varied practice.*** Opportunities to practice are important for supporting a student's fluency with computing, but not all opportunities to practice are equally effective. Varied practice is important. Rather than giving the student a set of nearly identical problems to do (which could develop automaticity), vary the types of problems within a single practice session to help build not only automaticity but also flexibility and knowledge of multiple and appropriate computational strategies. For example, distribute opportunities to practice a particular problem over time and interweave different types of problems together.
4. ***In a test-like situation, give the student the opportunity to recall and apply mathematics computational skills.*** This opportunity to practice can ultimately improve the student's ability to remember and perform computation strategies. The "testing effect" finds that in addition to giving students varied practice with using strategies, it also is important to provide opportunities to practice testing. This type of testing gives students the needed experience in recalling information

from memory. It also gives students a better understanding of what they have a firm grasp on and what they need more practice on.

5. ***Focus on growth over performance when implementing timed tests.*** It is common for tests of computation to be timed, with students asked to accurately perform as many computations as possible in a given period of time. However, the costs of timed drills to students' performance may outweigh their benefits, because timed tests tend to increase anxiety among students. High amounts of anxiety can decrease a student's working memory and attention, making it even more difficult for the student to focus on the problem and correctly solve it. An alternative to timed tests is to provide practice (or even practice tests) in a game-like atmosphere, such as with flashcards or apps. Furthermore, grading relative performance, such as focusing on individual growth over time rather than absolute number of problems answered, may alleviate some of the anxiety that these types of timed tests sometimes create. More generally, in all assessment situations, instead of publicly ranking students in terms of who is performing better than others, a teacher is likely to be more effective in promoting greater student effort by making students more aware of their own progress over time than by encouraging comparisons with others.
6. ***Increase the student's repertoire of procedures by analyzing different strategies for solving the same problem.*** To make strategic choices about which procedure to choose for solving a given problem, the student is likely to benefit from being exposed to a range of possible strategies and from considering why a specific strategy might be more or less appropriate for a problem.
7. ***Implement number talks to expose the student to a range of computation strategies.*** "Number talks" are an informal, ungraded context where students can talk through their reasoning for using a certain procedure to solve a given problem. During a number talk, students solve a problem mentally and take turns sharing aloud how they thought about the problem while their teacher records the students' thinking and asks probing questions. Students are encouraged to find more than one strategy, and they are given individual thinking time to generate multiple strategies before sharing aloud. Students are asked to defend and justify their solutions and try to understand their peers' strategies.
8. ***Have the student interpret worked examples to increase his or her repertoire of mathematics strategies.*** Comparison of solutions has been shown to be an effective way to promote procedural fluency (e.g., Star & Rittle-Johnson, 2009). Comparison involves presenting problems and solution methods side-by-side, which can give teachers an opportunity to discuss what's best and why. This exercise gives the student a chance to compare and contrast different solutions and to reflect on what each solution has to offer in relation to particular tasks. It also promotes flexible use of procedures.
9. ***Promote number sense through multiple representations.*** Particularly in the early grades, students need opportunities to develop a sense of a number's quantity and magnitude. Instruction that provides multiple opportunities for them to engage with representations emphasizing quantity and magnitude help support the development of number sense. For example, linking a pictorial representation of an amount of concrete objects to the corresponding numerical symbol can be useful for promoting early number sense. In another example, estimating the magnitude of a number with number lines is also helpful in developing number sense. This practice can be incorporated into a game such as the board game *The Great Number-Line Race* (http://www.interventioncentral.org/academic-interventions/math/number-sense-promoting-basic-numeracy-skills-through-counting-board-ga-0).
10. ***Ask the student to reflect on whether the answers to computational problems are reasonable.*** When you ask students to explain whether and why their answers make sense, they may be more likely to evaluate the reasonableness of their answers on their own or future problems. Encourage the student to estimate and think about whether his or her answer makes sense.
11. ***Have the student reflect on mathematical errors.*** Reflecting on mathematical errors helps students understand why procedures work. Students benefit from analyzing a strategically designed incorrect solution that depicts a common error. Reflecting on errors increases a student's attention to important mathematical features of a problem type, which can then improve later performance on similar problems. This approach involves asking the student to study an error in an incorrect

worked example that has been pointed out to the student or class. After reflecting on the error, the student is asked to correct that error and then solve a similar problem to practice. A step-by-step guide to creating incorrect worked examples can be found in McGinn, Lange, and Booth (2015). In another example, teachers can choose a common error and then talk the class through the thinking behind the error. For a video-recorded example, see My Favorite No: Learning from Mistakes (https:/www.teachingchannel.org/videos/class-warm-up-routine). In general, reflecting on common errors can support improvements in both fluency and understanding of procedures, particularly among students who have low prior knowledge or are in the early stages of skill acquisition (Barbieri & Booth, 2016).

12. ***Value a range of ways to demonstrate competence in mathematics.*** To create classrooms where more students can see themselves as capable of doing mathematics, teachers and schools need to recognize more capabilities beyond getting a correct answer.

RECOMMENDED RESOURCES

Websites

https://ies.ed.gov/ncee/wwc/FWW/Results?filters=,Math

This website from the U.S. Department of Education provides links to information about evidence-based instructional interventions in mathematics.

http://mathapps.kent.edu/

This searchable website provides apps for mathematics learning, many of which can be used to support computational practice and some that also support development of meanings underlying procedures.

Books

Humphreys, C., & Parker, R. (2015). *Making number talks matter: Developing mathematical practices and deepening understanding, Grades 4–10*. Portland, ME: Stenhouse.

This book provides information on incorporating number talks into mathematics instruction to support the development of a repertoire of solution strategies and an understanding of why procedures work.

Jordan, N. C., & Dyson, N. (2014). *Number sense interventions.* Baltimore, MD: Brookes.

This book provides a series of evidence-based lessons to improve early number sense. Designed for kindergartners who struggle with numbers, number operations, and number relations, the 24 lessons cover skills such as oral counting, number recognition, and numeral writing. Black-line masters of lesson materials are included for easy photocopying.

Related Helping Handouts

Engagement and Motivation: Helping Handout for School

Mathematics Problem Solving: Helping Handout for School

Math Skills: Helping Handout for Home

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