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Mathematics Problem Solving: Helping Handout for School

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INTRODUCTION

Problem solving in mathematics goes beyond finding answers. It involves students' mathematical knowledge, students' strategies for solving problems, students' use of their knowledge and strategies, and students' view of themselves as doers of mathematics. The best way for students to improve at problem solving is to have more opportunities to solve challenging mathematical problems with appropriate support. Mathematical problem solving is an area in which a student can have a specific learning disability (the other is math computation). However, students with a learning disability are not the only ones who might struggle with math problem solving. The recommendations in this handout are useful for teaching math problem solving to all students, but they especially target students who are experiencing learning difficulties in math.

WHAT TO CONSIDER WHEN SELECTING INTERVENTION STRATEGIES

To help students struggling with problem solving in mathematics, teachers should assess, either formally or informally, which factors might contribute to a student's difficulties. These factors can include whether and how the student is challenged by comprehending the problem, whether the student has limited capabilities in selecting and using a wide repertoire of strategies, or whether the student is reluctant to persevere in the face of a difficult problem. It is important to recognize that what can look like a student's unwillingness to persevere may actually be the student having difficulty with comprehension or having a limited repertoire of strategies to employ. Teachers should identify and respond to challenges that individual students are facing, and also determine how to adjust classroom instruction to create an environment where students who struggle with math problem solving can be more likely to succeed.

A clinical interview is a powerful approach to identify the difficulties a student is facing with problem solving in mathematics. Prior to the interview, the teacher designs math problems and writes interview questions to explore conjectures about what is difficult for the student. During the clinical interview, the student is presented with a mathematical task similar to one that seems challenging for the student, and the student is asked to think aloud while solving it. The purpose of the interview is to examine whether the conjecture about the student's difficulties is correct or if the student has other additional difficulties.

In a clinical interview, the teacher asks a student open-ended prompts to elicit information about how the student is thinking, such as "Say more, please," and "How do you know?" or "How did you decide?" The interaction is nonevaluative. Targeted prompts can be designed to elicit particular aspects of students' thinking, such as "Draw a picture of what is happening in this problem."

A clinical interview can be used to diagnose a range of difficulties. A teacher could ask herself or himself the following questions about the challenges the student is facing, to develop conjectures about the student's thinking to explore in the interview:

- Is the student having difficulty with reading comprehension, such as for identifying the goal of the problem and the operations needed to solve the problem? If the problem is a contextualized story problem, the student may be able to execute the mathematical calculations necessary to solve

the task, but may struggle to understand what the task requires. Students may look for numbers and key words and overgeneralize relationships in problems incorrectly. They also may try to make computations without making connections to the problem.

- Does the student have a limited repertoire of strategies for solving math problems? Students may struggle with math problems that look different from examples they have worked on in class. Students may know only one strategy for solving math problems. They may be unsure about possibilities for approaching the problem, such as how to use visual representations for interpreting and solving tasks.
- Is the student willing to persevere in the face of challenge? Students can lack stick-to-itiveness when doing mathematics problem solving. Students who lack perseverance can have a fixed mind-set toward math. They may think that some people are born good at math while others are not. Students may decide that a problem is too difficult and then stop. They may not hold a growth mind-set; that is, they do not recognize that, with effort, everyone can improve their math problem solving (Dweck, 2010).

Once the clinical interview reveals which of these or other difficulties the student is facing, then instructional approaches could be implemented that address the student's particular challenges.

RECOMMENDATIONS

Teachers can use these recommendations preventively or as an intervention for students struggling with math problem solving. The recommendations are grouped into three categories: (a) scaffolding comprehension of problem-solving tasks, (b) increasing the repertoire of problem-solving strategies, and (c) supporting perseverance.

Scaffolding Comprehension of Problem-Solving Tasks

1. ***Implement scaffolding strategies that help the student develop and apply metacognitive skills to math problem solving.*** One example is the I-THINK framework (Lynch, Lynch, & Bolyard, 2013), which prompts the student to reflect on how to solve the problem: ***I****ndividually think about the problem.* ***T****alk about the problem.* ***H****ow can it be solved?* ***I****dentify a strategy to solve the problem.* ***N****otice how your strategy helped you solve the problem.* ***K****eep thinking about the problem. Does it make sense? Is there another way to solve it?* This set of prompts slows down the problem-solving process to engage the student in reflecting on strategies that can be used and on why they make sense. The student also can benefit from using a journal to self-monitor by creating and following personal reflection prompts.
2. ***Scaffold the student's comprehension by reading through a contextual story problem in phases, using a "three-reads launch."*** In a three-reads launch, students take time to make sense of mathematical relationships in a problem, thinking in phases, prior to solving it. The teacher projects or displays a story problem stem without including a question for students to answer. Next, students read the problem stem out loud three times. The teacher may rotate through individual readers or use choral reading. After each read, the teacher asks the students a reflection question. After the first read, the teacher asks students to reflect on the problem context—What is happening in this situation? After the second read, the teacher asks students to identify the quantities in the problem. After the last read, the teacher asks students to determine what questions they could ask and answer with the given quantities for that situation. Students pose the problem together as a class. This approach supports emerging bilingual students, students with learning disabilities in literacy, and all learners who might otherwise get ahead of themselves when reading story problems.
3. ***Ask the student to brainstorm what he or she notices and wonders about the problem stem prior to solving it.*** This can be done with the individual student or with the entire class. A noticing and wondering protocol allows students to attend to the meanings within the task prior to solving it. When posing a problem, the teacher asks the student, or class, two questions: What do you notice? What do you wonder? By asking these questions, the teacher promotes the students' understanding of the problem statement and piques their interest in the problem. Students benefit from sharing what they notice and wonder. The teacher can ask the class to share in an open brainstorming session or they can have students first write and reflect individually about what they notice or wonder about a problem. The important

feature of the noticing and wondering protocol is that students conduct this reflection prior to solving the problem.

4. ***Change the problem-solving context to something that is more familiar to the student.*** Students who struggle to identify and make sense of the quantitative relationships in a story problem may lack experience with the contextual features in the problem-solving situation. For instance, if a student does not have experience with a problem involving sales tax (e.g., your school is in Delaware or Oregon, with no sales tax), use a different problem involving percentage increase. See if changing the situation to align more closely with the student's lived experiences makes the mathematical relationships in the problem more accessible. Keep the student's home culture in mind when making these adjustments.

Increasing the Repertoire of Problem-Solving Strategies

5. ***Select mathematics problems that invite a wide range of student thinking, such as problems that can be viewed as low floor, high ceiling tasks.*** Such problems are accessible to students with different levels of mathematical knowledge, because the low floor provides many different entry points into the problem. The high ceiling gives room for exploration and challenge.
6. ***Help the student build an association between the problem type and the solution strategy.*** Give the student opportunities to classify mathematical tasks as similar and different. Help the student name and label similar problems, such as those with the same problem structure and operations required for solving. Then when approaching future problems, the student can consider whether and in what ways a problem looks like one encountered previously. The student can reflect on how the prior problem was solved and apply that strategy to the new problem.
7. ***Ask the student to interpret incorrect strategies used by peers.*** Display an incorrect strategy used by a fictional or anonymous student in the class. Ask the class to interpret that student's problem-solving work. Have the class discuss the solution in a way that allows the students to become the authorities when assessing the correctness of a situation, rather than the teacher or adults always being the authorities. Tell the class that the strategy is incorrect; then ask the student to explain how the solution started positively and where the solution fell short. Then ask the student to explain the error and say why it is an error. Finally, ask the student to correct the error and explain why the change in the strategy is an improvement.
8. ***Encourage the student to explain relationships between two correct strategies.*** If there is more than one way to correctly solve a problem, present the student with two correct strategies. Ask the student to determine why each strategy works. Then ask the student to identify ways that the two strategies are different and ways they are similar. Present the student with another problem that can be solved using both strategies and ask the student to try each strategy when solving it.
9. ***Support the student by using a diagram to illustrate mathematical relationships in a problem.*** Diagrams can be used to make sense of or help students see mathematical connections and structures in a problem. Introduce the class to a diagram and ask the student to explain how the diagram represents the mathematical relationships in the problem. The student also could draw a diagram to explain how it represents the problem. Also ask the student to interpret a diagram drawn by a peer.

Supporting Perseverance

10. ***Challenge the notion of what it means to "do mathematics."*** Students tend to get frustrated if they view the purpose of mathematics problem solving as primarily finding answers quickly, particularly if doing so is hard for them. Students work harder when they recognize that doing mathematics also includes explaining their thinking, defending whether something is true, noticing or recognizing a pattern, justifying and generalizing that pattern, drawing a diagram to illustrate a mathematical relationship, and noticing connections between different solution strategies, among other activities. By naming and labeling these mathematical activities, processes, and practices, teachers can help students become aware that mathematics is more than calculating and can reduce their frustration if a problem takes a while to solve.
11. ***Create a safe culture for taking intellectual risks by normalizing the struggle.*** In classrooms that overemphasize competition, some students are winners and others are losers. For example, some students may declare a task to be easy

while others have not finished it yet. Help students understand that the learning process is inherently a bumpy one for everyone. Promote the value of making mistakes and being uncertain when solving problems. After all, if students already knew how to do a problem, they would be reviewing the material, not learning something new. Normalizing the challenge of the learning process reduces the student's anxiety associated with taking a risk because it prevents students from thinking that there is something wrong with them specifically when they are struggling.

12. ***Promote rough-draft thinking to support the student with taking intellectual risks.*** Students may expect that sharing their thinking about their problem-solving process is a final-draft performance, saying what they already know. However, students can continue to learn by sharing their thinking during class discussions. If they share ideas that are in progress and if they are willing to revise their ideas through feedback, they will keep learning during the discussions. By naming and labeling this process of sharing ideas that are in progress as rough-draft thinking and by being explicit about why rough-draft thinking supports learning, you can foster a safer classroom culture.

RECOMMENDED RESOURCES

Websites

https://www.youcubed.org/tasks/

http://nrich.maths.org/

These two websites are good sources for low floor and high ceiling problem-solving tasks.

Books

Flynn, M. (2016). *Beyond answers: Exploring mathematical practices with young children.* Portland, ME: Stenhouse.

This book is geared toward supporting students in early elementary grades with problem solving in mathematics.

Kelemanik, G., Lucenta, A., & Creighton, S. J. (2017). *Fostering mathematical practices.* Portsmouth, NH: Heinemann.

The authors of this book provide guidance on developing instructional routines for secondary mathematics lessons that engage students in problem solving.

Ray-Riek, M. (2013). *Powerful problem solving: Activities for sense making with the mathematical practices.* Portsmouth, NH: Heinemann.

This book is a resource for mathematical tasks that promote understanding so that students learn through problem solving.

Related Helping Handouts

Engagement and Motivation: Helping Handout for School

Mathematics Computation: Helping Handout for School

Math Skills: Helping Handout for Home

REFERENCES

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